ENERGETIC EFFICIENCY OF TWO IMPELLERS ON THE SAME SHAFT IN A CYLINDRICAL BAFFLED VESSEL OF HIGH HEIGHT/DIAMETER RATIO*

Ivan FOŘT[a], Jiří HÁJEK[b] and Václav MACHOŇ[c]
[a] *Department of Chemical and Food Process Equipment Design,*
Czech Technical University, 166 07 Prague 6
[b] *Chemopetrol, Spolana, 277 11 Neratovice and*
[c] *Department of Chemical Engineering,*
Prague Institute of Chemical Technology, 166 28 Prague 6

Received November 16, 1988
Accepted January 24, 1989

The paper deals with the experimental study of the indicating particle circulation and the impeller power input in a liquid mechanically agitated with two high-speed impellers (combination of the standard turbine impeller and the six inclined (at 45°) plane blades impeller) on the same shaft in a slender vessel (its height is equal double of the vessel diameter) equipped with four radial baffles at its walls under the turbulent regime of agitated charge flow. By the visual method of the indicating particle it is examined its circulation in the lower part of the system (pumping effect of the lower impeller), its circulation in the upper part of the system (pumping effect of the upper impeller), and the exchangeable circulation between the upper and lower part of the system and vice versa. The impeller power input is ascertained from the measured current electricity in the anchor of the direct current driving motor. It follows from the calculated energetic efficiency (the ratio of the cube of the sum of the impeller flow rate numbers and the sum of the impeller power numbers) of the investigated combinations of impellers that the highest value of this quantity is exhibited for two standard turbine impellers on the same shaft and for a combination of the lower standard turbine impeller and the upper impeller with inclined plane blades pumping upwards; slightly less value of the impeller energetic efficiency appears for the combination of two impellers with six inclined plane blades, the upper one pumps liquid upwards and the lower one downwards. For all the configurations the vertical distance of impellers on the same shaft has to be longer than the vessel diameter.

In a number of areas and in particular in industrial fermentation systems liquid circulation takes place in fermentors having several impellers on the same shaft. While such devices are fairly common in practice, their optimization sofar has to rely on only a very modest theoretical background. Moreover, no sufficient theoretical description of power input or pumping effects of impellers is available for the systems with multiple impellers where the hydrodynamic conditions are much more compli-

---

* Part LXXVII in the series Studies on Mixing; Part LXXVI: Collect. Czech. Chem. Commun *54*, 1599 (1989).

cated than for the systems involving a single impeller. When attempting to optimize the equipment setup and to look for the scaling-up procedure the impeller power input is the primary information required. Nevertheless, it is very important to know the conditions under which the highest intensity of liquid circulation at the lowest impeller power input is reached. This is the major problem dealt with this paper.

Considerable attention has been paid in the literature[1-4] to the impeller power input in the system with multiple impellers both at aerated and at non-aerated conditions under the turbulent regime of the charge flow. When two impellers on the shaft were used the combinations of two standard (Rushton) turbines and/or a standard turbine impeller (a lower impeller) and impeller with inclined blade impeller (an upper impeller) pumping liquid upwards or downwards were used. It follows from published data that, approximatelly, the impeller power input of two impellers on the same shaft when the vessel height diameter ratio is equal two can be considered as a sum of power inputs of two single impellers when the vessel height/ /diameter ratio is equal one, if the distance between impellers is greater than the diameter of the vessel. Similar conclusion follows from the observation of the liquid circulation in the vessel with two impellers on the same shaft[5] under the turbulent regime of the charge flow. The flow rate numbers of both impellers (the upper and the lower ones) do not depend on their mutual vertical distance if this distance is longer than the diameter of the vessel. However, the values of the flow rate numbers are greater than the values of these criteria determined in agitated systems with single impellers when the vessel height/diameter ratio is equal one. Both the power number and the flow rate number does not depend on the mixing Reynolds number for the agitated systems of the high height/diameter ratio when mixing Reynolds number is greater than five thousand.

A simple view on the effect of rotary impellers can be obtained by determination of either their power input or their intensity of circulation. Combinatiin of both the above mentioned criteria, the impeller energetic efficiency[6], gives more reliable view on their optimization with respect to their ability to form an intensive circulation of agitated charge at the lowest values of the impeller power input.

THEORETICAL

The aim of this study is to investigate an arrangment with dual impellers to reach the maximum value of the impeller energetic efficiency of the upper impeller (index I) and the lower impeller (index II) on the same shaft (Fig. 1)

$$E \equiv (Kp_{\mathrm{I}} + Kp_{\mathrm{II}})^3/Po\,. \qquad (1)$$

Then the impeller power number

$$Po = \frac{P_{\mathrm{I}} + P_{\mathrm{II}}}{\varrho n^3 d^5} \qquad (2)$$

and the impeller flow rate numbers

$$Kp_i = \dot{V}_i/nd^3 \,, \quad (i = \mathrm{I}, \mathrm{II}) \qquad (3)$$

of both impellers has to be determined.

We consider the cylindrical agitated system with flat bottom and four radial baffles of width b at its walls. The height of agitated liquid at rest H above the vessel bottom is equal to double of the vessel diameter D. Two types of impellers are used for mixing: standard (Rushton) turbine impeller[7] and/or blade impeller with six inclined plane blades ($h = 0{\cdot}2d$, $\alpha = 45°$) (ref.[7]) pumping liquid downwards or upwards. Four arrangements of the mutual impeller positions (Table I) are considered the distance between the upper and the lower impellers Δc is varying. The liquid circulation takes place under its turbulent regime of flow.

The agitated volume is divided into two parts (see Fig. 1): Region I corresponds to the volume arround the lower impeller to the height $H_2 + \Delta c$ and region II corresponds to the remaining volume of charge arround the upper impeller. For the two regions four transitions (circulations) of the indicating particle are defined: transition I – circulation between two succesive passages of the indicating particle through the rotor region (i.e. the region sorrounding the rotating impeller) of the lower impeller without passing through the rotor region of the upper impeller; transition II – circulation between two successive passages of the indicating particle through the rotor region of the upper impeller without passing through the rotor

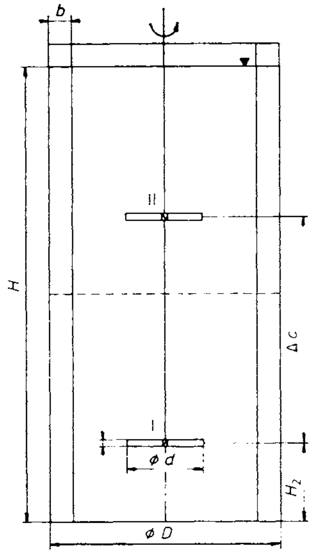

FIG. 1

Agitated system with dual high-speed impellers region of the lower impeller; transitions I–II and II–I – circulation of the indicating particle between the rotor regions of the lower and upper impeller (I–II) or the other way round (II–I). The measured circulation time of indicating particle is considered in the following way:

1) Total time of circulation θ_t – time taken in measuring the circulation.
2) Total time of motion of the indicating particle in the single region $\theta_{t,i}$ ($i =$ I, II, I–II, II–I).
3) Average time of circulation of the indicating particle in the single region

$$\theta_{av,i} = \theta_{t,i}/m_i\,, \quad (i = \mathrm{I, II, I{-}II, II{-}I})\,. \tag{4}$$

The following relations hold between the circulation quantities $\theta_{t,i}$ and $\theta_{av,i}$:

$$\theta_t = \sum_i m_i \theta_{av,i}\,, \quad (i = \mathrm{I, II, I{-}II, II{-}I}) \tag{5}$$

and/or using Eg. (*4*)

$$\theta_t = \sum_i \theta_{t,i}\,, \quad (i = \mathrm{I, II, I{-}II, II{-}I})\,, \tag{5a}$$

where m_i is the number of single transitions.

The circulation time of liquid particle is determined by the length of circulation loop and by velocity at each its point. Since the liquid velocity differs at each point of agitated charge and each circulation loop is of different length, the time of circulation has a certain distribution characterizing the circulation flow[8]. For each system

TABLE I
Arrangement of impeller positions on the same shaft

Arrangement	Lower impeller[a]	Upper impeller
A	standard turbine impeller	standard turbine impeller
B	standard turbine impeller	impeller with inclined plane blades pumping downwards
C	standard turbine impeller	impeller with inclined plane blades pumping upwards
D	impeller with inclined plane blades pumping downwards	impeller with inclined plane blades pumping upwards

[a] The lower impeller is always located at $H_2 = D/3$ off-bottom

region its volume V_i $(i = \text{I, II, I–II, II–I})$ is defined. The relations for arrangements A, B and C are given in Table I of the cited work[5], the relations for arrangement D are given in Table II. For the turbulent flow of agitated charge, when the Reynolds number for mixing $Re_M > 5 \,.\, 10^3$ and for greater values of the quantity Δc than certain critical value, it is possible to assume that the region volumes considered are fully filled-up with circulating charge – the existence of dead (non-agitated) regions in the system is not considered. Then the volumetric flow rate through the i-th region can be determined from the relation

$$\dot{V}_i = V_i/\theta_{av,i}\,, \quad (i = \text{I, II, I–II, II–I})\,. \tag{6}$$

The above mentioned characteristics of circulation in the system investigated can be expressed in the dimensionless form:
flow rate number

$$Kp_i \equiv \dot{V}_i/nd^3\,, \quad (i = \text{I, II, I–II, II–I}) \tag{7}$$

and the relative time of the particle stay in region i

$$\theta_{t,i,rel} = \theta_{t,i}/\theta_t\,, \quad (i = \text{I, II, I–II, II–I})\,. \tag{8}$$

All the quantities said depend on the impellers distance Δc which can be expressed in dimensionless form as $\Delta c/d$.

EXPERIMENTAL

The principle of the method of visual observation of motion of the indicating particle[9] in charge and storage of its passages through the rotor regions of single impellers was described in the cited work[5]. The above mentioned circulations (transitions) I, II, I–II and II–I were observed and corresponding times of circulations (transitions) were measured. Then within the framework of one experiment (i.e. for the given arrangement of the agitated system (see Table I), value of Δc and level of frequency of impeller revolution n) we obtained the values of $\theta_{av,i}$ and m_i.

The values of the dual impellers power input were ascertained by determining the current electricity in the anchor I_k of the direct current motor which tends to be directly proportional to the electromagnetic moment of the motor. The torque moment at the motor shaft is smaller

TABLE II

Relations for calculations of volumes V_i of regions in agitated system for arrangement D

i:	I	II	I–II	II–I
V_i:	$0{\cdot}25\pi D^2(H_2 + \Delta c/2)$	$0{\cdot}25\pi D^2(H - H_2 - \Delta c/2)$	$0{\cdot}25\pi D^2(H_2 + \Delta c/2)$	$0{\cdot}25\pi D^2(H - H_2)$

by the loss moment M_z and the impeller power input P can then be ascertained from the equation

$$P = 2\pi K_e\, n(I_k - I_{ko})\,, \qquad (9)$$

where I_{ko} is the current electricity passing through the anchor of the unburdened motor, n the frequency of impeller revolution and K_e the motor constant found by independent measurements.

The measurements were carried out in a cylindrical vessel (see Fig. 1) of inside diameter $D = 0{\cdot}29$ m and filling height $H = 0{\cdot}58$ m, filled with tempering water or water–glycerol solution, located in a square vessel of inside edge length $a = 0{\cdot}396$ m, filled with tempering water. Both vessels were made of perspex. Along the whole height of the cylindrical vessel, four radial baffles of width $b = 0{\cdot}1D$ were located symmetrically. In the vertical axis of symmetry of the vessel, dual impellers were located in the arrangement D (measurement of the indicating particle circulation as well as measurement of the dual impellers power input) and in the arrangements A, B, or C (measurement of the dual impellers power input). Diameter of all the impellers used was $d = 0{\cdot}097$ m; the ratio $d/D = 1/3$. The vessel was filled-up with aqueous or glycerol aqueous NaCl solution of density $\varrho = 1\,044$ kg/m^3 equal to the density of material of indicating particle used. The temperature at which the measurement was taken was within the range 20—22°C, the impeller frequency of revolution n was set within $2{\cdot}5 - 7{\cdot}5\ \mathrm{s}^{-1}$ and it was measured by means a photoelectric senzor. The measurements were performed within the impeller distance $\Delta c \in \in \langle 1{\cdot}2d;\ 3d \rangle$. For the given value of Δc the impeller power input was measured at eight elvels and the indicating particle circulation was measured at three levels of the impeller frequency of revolution. Setting the lowest value was conditioned by the turbulent regime of charge.

The dimensions of impellers and vessel, as well as the distance between the impellers and liquid level and bottom were measured with a milimeter scale with the accuracy of $\pm 5\,.\,10^{-4}$ m. The impeller frequency of revolution was measured with a photoelectric senzor: The relative measuring accuracy of this quantity reached at most $\pm 1{\cdot}6\%$ of the set value of frequency. The density of charge in investigations performed was within 1 043—1 044 kg/m^3, its viscosity within the range $1{\cdot}05 - 4{\cdot}8\,.\,10^{-3}$ Pa s. The dependent quantities were measured with following accuracies: Time of circulation was measured with the accuracy of experimenter $\pm(0{\cdot}2 - 0{\cdot}3)$ s, the motor constant K_e (see Eq. (9)) was determined with the relative accuracy of $\pm 0{\cdot}5\%$, the difference $(I_k - I_{ko})$ was determined with the accuracy $\pm 0{\cdot}03$ A. With respect to the loss moment M_z the lowest value of the impeller frequency of revolution taken into consideration for measurement of the impeller power input was $n = 5\ \mathrm{s}^{-1}$, i.e. $Re_M = 6{\cdot}0\,.\,10^4$ (see Fig. 2). Then the calculated value of the power number does not depend on the Reynolds number for mixing if air is not absorbed through the liquid surface into the agitated charge.

During the whole series of measurements, the value of the mixing Reynolds number was within $Re_M \in \langle 5{\cdot}23\,.\,10^3;\ 7{\cdot}02\,.\,10^4 \rangle$ so that the flow regime of agitated charge was therefore turbulent.

RESULTS AND DISCUSSION

The impeller power number of dual impellers does not depend on the Reynolds number for mixing at the turbulent regime of flow (see Fig. 2). This fact corresponds to the results published earlier[1-4]. However, in certain arrangements there is a dependence between the impeller power number and the relative impeller distance (Fig. 3). For arrangement A (two standard turbine impellers) the impeller power number decreases within the limits $\Delta c/d \in \langle 2{\cdot}0;\ 1{\cdot}6 \rangle$. Above and below this interval quantity Po is constant. These results fairly agree with the published data[1], as well as, with the jump of the dependence $Kp_{\mathrm{i}} = Kp_{\mathrm{i}}(\Delta c/d)$, $(i = \mathrm{I, II})$ for arrangement A (ref.[5]). Here the flow rate numbers Kp_{i}, $(i = \mathrm{I, II})$ for both impellers change significantly their values within the interval $\Delta c/d \in \langle 1{\cdot}8;\ 2 \rangle$. This fact can be accounted for considering that with a sufficient distance of the impellers, their flow patterns do not influence and are the same for each impeller as in case of flow patterns with one impeller. In case of smaller distance between the impellers, the streams streaking from the single impellers influence each other. Consequently, the values of quantities Kp_i, $(i = \mathrm{I, II})$, as well as, of quantity Po decrease in comparison with the previous state. The same explanation can be expected for arrangement D (two impellers with inclined plane blades), however, the continuous dependence of the power number on

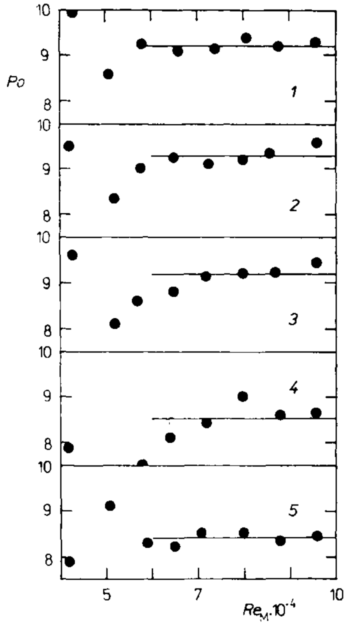

FIG. 2

Dependence of the power number of dual impellers on impellers frequency of revolution for arrangement A. *1* ($\Delta c/d = 3$, $Po = 9{\cdot}2$, $\sigma_{\mathrm{Po}} = 0{\cdot}123$); *2* ($\Delta c/d = 2$, $Po = 9{\cdot}3$, $\sigma_{\mathrm{Po}} = 0{\cdot}21$); *3* ($\Delta c/d = 1{\cdot}8$, $Po = 9{\cdot}2$, $\sigma_{\mathrm{Po}} = 0{\cdot}21$); *4* ($\Delta c/d = 1{\cdot}4$, $Po = 8{\cdot}55$, $\sigma_{\mathrm{Po}} = 0{\cdot}31$); *5* ($\Delta c/d = 1{\cdot}2$, $Po = 8{\cdot}42$, $\sigma_{\mathrm{Po}} = 0{\cdot}14$)

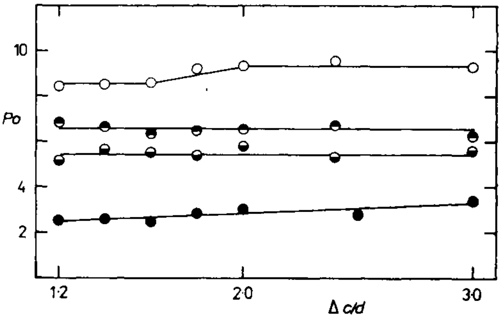

FIG. 3

Dependence of the power number of dual impellers on the relative impeller distance. ○ Arrangement A, ◒ arrangement C, ◓ arrangement B, ● arrangement D the impeller distance characterises the continuous change of the flow pattern in the system with dual impellers. For arrangements B and C (standard turbine impeller and impeller with inclined plane blades) no significant change of the impeller power input appears within the limits $\Delta c/d \in \langle 1 \cdot 2; 3 \rangle$. However, the direction of flow pumping by the upper (plane blades) impeller effects on the power input of dual impeller. The more the pumping effect of both impellers, the more their power input. Nevertheless, in all cases investigated the power input of dual impellers is slightly less than the sum of the power inputs of single impellers[7] in the system where height of liquid H is equal to vessel diameter D. This fact can be accounted for considering the absence of the solid vessel bottom bellow the upper impeller. Then the friction losses in the horizontal plane separating the circulation regions of both impellers decrease and the power input of the upper impeller, as well. The values of the power input of dual impellers published in cited work[2] measured by balancing the rotational movement of the vessel for arrangements A, B and C agree fairly well with the data of this study. The largest relative difference amounts to 10%.

Flow rate number Kp_{I} for the upper impeller in arrangement D is approximately equal to the flow rate number for single impeller[9] and quantity Kp_{II} for the lower impeller in the given arrangement (see Fig. 4) is approximately equal to 1·5 – fold the flow rate number for single impeller[9] in case of the system where $H = D$. When approaching the impellers, values Kp_{II} and $Kp_{\text{II}-\text{I}}$ increase with decreasing value of simplex $\Delta c/d (\Delta c/d < 1 \cdot 8)$. On the contrary values Kp_{I} and $Kp_{\text{I}-\text{II}}$ are constant in the whole interval of simplex $\Delta c/d$ investigated. This phenomenon can be explained by the nonvalidity of the simplifying assumption that no dead (non-agitated) regions exist in the system. Circulation loops in the upper part of charge (regions V_{II} and $V_{\text{II}-\text{I}}$) do not reach the whole volumes of these regions (see Fig. 5) and so times of circulations $\theta_{\text{av},\text{II}}$ and $\theta_{\text{av},\text{II}-\text{I}}$ measured visually do not correspond to the sizes of the regions V_{II} and $V_{\text{II}-\text{I}}$, respectively (Table II). Then quantities $\dot{V}_{\text{II}}$ and $\dot{V}_{\text{II}-\text{I}}$, or Kp_{II} and $Kp_{\text{II}-\text{I}}$ are overstimated. Flow characteristics of the lower loops Kp_{I} and $Kp_{\text{II}-\text{I}}$, on the contrary, do not depend on quantity $\Delta c/d$ over the whole interval of the simplex investigated.

The relative time of stay of the indicating particle in region I decreases with decreasing value of Δc, in region II and, on the contrary, increases with decreasing value of Δc (the off-bottom distance H_2 of the lower impeller not being changed) (see Fig. 6), the sum of these relative times not being dependent on the impeller distance. This effect is to be accounted for considering that the upper impeller with pumping effect upwards entrains the liquid from region I to region II. This entrainment is due to the coincident direction of flow at the boundary of both these regions, which, after all. contributes to the decrease of friction losses between the circulating flows at the given boundary surface (see Fig. 5). As to the properties of flow pattern, system D is similar to system C with the only one significant difference: the relative stay of the indicating particle in regions I and II is for system D much more higher

(83%) than for system C (65%). Two axial impellers exhibit less mutual interactions than the combination of the radial (turbine) and axial impellers which can be suitable for mechanically agitated bioreactors with shear sensitive supensions of microorganisms[10].

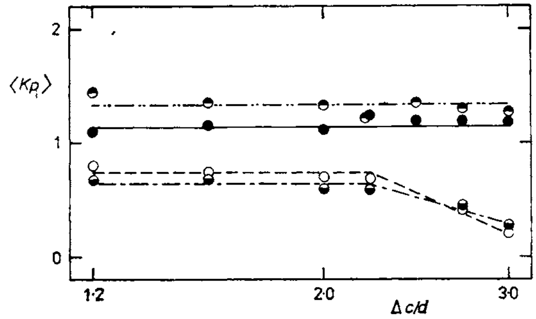

FIG. 4

Dependence of the flow rate number on the relative impeller distance for arrangement D. i = I (●; ———), II-I (◒; — · · —), I-II (◓; — · —), II (○; — — —)

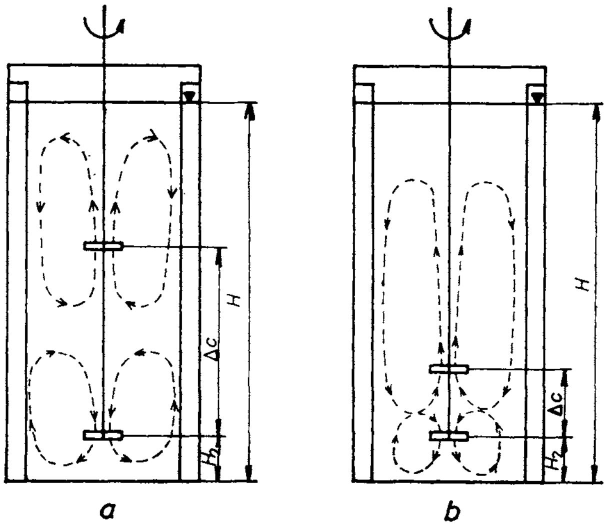

FIG. 5

Flow patterns for arrangement D: *a* $\Delta c/d = 3$, *b* $\Delta c/d = 1{\cdot}5$

Energetic efficiency of impellers is directly proportional to their hydraulic efficiency[6]. Therefore, the higher the impeller energetic efficiency, the better the utilization of the impeller power input for the intensity of liquid circulation. Dependence of quantity E, calculated from the measured quantities Kp_{I}, Kp_{II} and Po according to Eq. (*1*) on the impeller distance (see Fig. 7) shows for all arrangements investigated two main parts: at higher values of simplex $\Delta c/d$ it appears the constant values of impeller energetic efficiency E and below certain critical values of quantity $\Delta c/d$ there is the step change of quantity E. This increasing part of dependence $E = E(\Delta c/d)$ corresponds to the above mentioned conditions when the dead (non-agitated) regions in volumes V_{II} and $V_{\mathrm{II-I}}$ appear and, then, it cannot be taken into consideration for comparisons of the arrangements investigated.

The first conclusion following from the results is to place the dual impellers on the same shaft to the appropriate distance when they do not influence each other. The second conclusion deals with the fact that arrangement A and C can be considered at the highest level of the impeller energetic efficiency so that they can be considered as the best configurations to ensure the maximum intensity of circulation at the lowest mechanical energy input. On the contrary, arrangement B seems to be the worst configuration. Arrangement D exhbits an intermediate value of the impeller energetic efficiency, but its applications for many fermentation media (e.g. for suspensions of moulds, animal/plant cells etc.) select this configuration as a very suitable arrangement of the dual impellers on the same shaft in the high height/diameter ratio agitated systems.

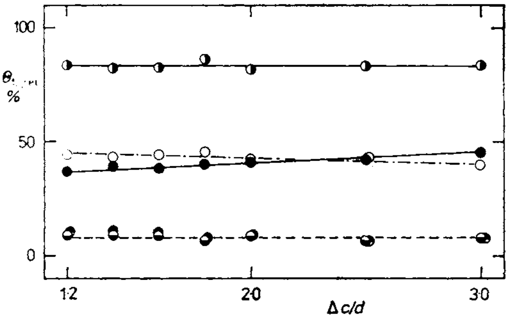

FIG. 6

Dependence of the relative time of stay of the indicating particle in the given region of the system on the relative impeller distance for arrangement D. i = I (●; ———) II-I (◒; – – –), I-II (◓; – – –), II (○; – · –), I+II (◑; ———)

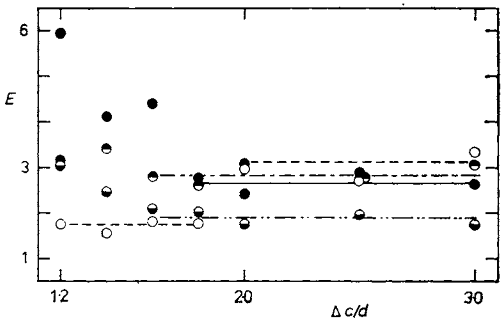

FIG. 7

Dependence of the impeller energetic efficiency on the relative impeller distance. Arrangement A (○; – – –), arrangement B (◒; – · · –), arrangement C (◓; – · – · –), arrangement D (●; ———)

LIST OF SYMBOLS

a	length of inside edge of square vessel, m
b	width of radial baffle, m
Δc	(mutual) vertical distance of impellers, m
D	inside diameter of cylindrical vessel, m
E	impeller energetic efficiency
d	impeller diameter, m
H	filling height of vessel, m
H_2	off-bottom distance of lower impeller, m
h	height of impeller blade, m
I_k	current electricity in the anchor of the direct current motor, A
I_{ko}	current electricity passing through the anchor of the unburdened motor, A
K_e	motor constant
Kp	flow rate number
m	number of passages of indicating particle through chosen volume
n	impeller frequency of revolution, s^{-1}
P	impeller power input, W
Po	power number
$Re_M = nd^2\varrho/\eta$	mixing Reynolds number
V	volume, m^3
$\dot{V}$	volumetric flow rate, $\mathrm{m}^3\,\mathrm{s}^{-1}$
η	dynamic viscosity, Pa s
θ	time of circulation, s
σ_{Po}	standard deviation of the power number
ϱ	density, $\mathrm{kg\,m}^{-3}$

Subscripts and Designations

av	average value in set
i	referred to i-th region of system
t	referred to whole
I	referred to lower impeller
II	referred to upper impeller
I–II	referred to transition between lower and upper impeller
II–I	referred to transition between upper and lower impeller
$\langle\ \rangle$	arithmetic mean of values over varying impeller frequency of revolution

REFERENCES

1. Hudcová V., Nienow A. W., Machoň V.: *Paper presented at the 9th International CHISA Congress, Prague 1987.*
2. Machoň V., Vlček J., Skřivánek J.: *Proceedings of the 5th European Conference on Mixing, Würzburg 1985,* p. 155.
3. Machoň V., Vlček J., Hudcová V.: *Proceedings of the 6th European Conference on Mixing, Pavia 1988*; p. 351.
4. Abrardi V., Rovero G., Sicardi S., Baldi G. and Conti R.: *Proceedings of the 6th European Conference on Mixing, Pavia 1988*; p. 329.

5. Fořt I., Machoň V., Hájek J., Fialová E.: Collet. Czech. Chem. Commun. *52*, 2640 (1987).
6. Fořt I., Medek J.: *Proceedings of the 6th European Conference on Mixing, Pavia 1988*; p. 51.
7. *Czechoslovak Standard Mixing Equipment 6910.* Výzkumný ústav chemických zařízení CHEPOS, Brno 1969.
8. Kudrna V., Koza V., Hájek J.: Collect. Czech. Chem. Commun. *54*, 633 (1989).
9. Fořt I. in: *Mixing: Theory and Practice* (V. W. Uhl and J. B. Gray, Eds), Vol. III, Chapter 14. Academic Press, New York 1986.
10. Meyer H. P.: Swiss Biotech. *6* (Nr. 4) 27 (1988).

Translated by the author (I. F.).